4	Work Power and Energy	Book:

Key equations and formulae

- **Work done,** $W = F.s$

  $W = \vec{F}.\vec{s} \cos\theta$

  Where F is force, s is displacement and θ is angle between them.
- **Work done by variable force**

  $W = \vec{F}.\vec{ds}$
- **Kinetic energy, K.E.** $= \frac{1}{2} mv^2$

  Where m = mass of body

  v = velocity of body
- **Work energy theorem**

  $W = \Delta K$

  ΔK = Change in kinetic energy
- **Gravitational potential energy**

  $U = mgh$

  Where, m = mass

  g = acceleration due to gravity

  h = height above ground
- **Potential energy of a spring,** $U = \frac{1}{2} kx^2$

  Where, k = spring constant

  x = extension is the spring
- **Power,** $P = \frac{W}{t}$

  $P = F.v$

  Where, F = force, v = velocity, W = work, t = time
- **Elastic collision in 1–D**

  $$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

  $$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \left(\frac{2m_1u_1}{m_1 + m_2}\right)u_1$$

1 Mark Questions

Q 1. A body of mass 20 kg is initially at a height of 3 m above the ground . It is lifted to a height of 2 m from that position. Its increase in potential energy is

(A) 100 J (B) 392 J

(C) 60 J (D) – 100 J

Sol. Option (B) is correct.

Q 2. Two masses 1 g and 4 g are moving with equal kinetic energies. The ratio of the magnitudes of their linear momenta is

(A) 4 : 1 (B) 1 : 2

(C) 0 : 1 (D) 1 : 6

Sol. Option (B) is correct.

Q 3. An electric heater of rating 1000 W is used for 5 hrs per day for 20 days. The electrical energy utilised is

(A) 150 kWh (B) 200 kWh

(C) 100 kWh (D) 300 kWh

Sol. Option (C) is correct.

Q 4. There is a person with a load on his head walking with uniform velocity on a horizontal road. Does he do any work ?

Sol. No, he is not doing any work. In order to overcome the load, the person exerts a force against gravity and this force is perpendicular to the direction of displacement. Hence the work done is $W = \vec{F}.\vec{S} = FS \cos 90° = 0$.

Q 5. Can kinetic energy of a body be negative ?

Sol. The kinetic energy ($1/2mv^2$) cannot be negative as both m and v^2 are always positive.

Q 6. Can potential energy of a body be negative ?

Sol. The potential energy can be negative when the force involved is of attraction.

Q 7. In which type of motion momentum changes but kinetic energy does not ?

Sol. In uniform circular motion momentum changes but kinetic energy does not change.

Q 8. In a perfectly inelastic collision is whole of the kinetic energy lost ?

Sol. No, only as much kinetic energy is lost as it is necessary for the conservation of linear momentum.

2 Marks Questions

Q 1. Explain positive work and negative work ?

Sol. If the force applied on a body is in the same direction as the displacement (motion) of the body, the work done by force is said to be positive ($W = Fs \cos 0° = Fs$). If the force acts opposite to the direction of motion, the work done by the force is said to be negative ($W = Fs \cos 180° = -Fs$).

Q 2. If two protons are brought towards each what will happen to their potential energy ?. If a proton and an electron be brought nearer, then ?

Sol. The potential energy will increase; because in bringing the protons nearer, work will be done against the force of repulsion which will be stored in the form of potential energy. The potential energy will decrease when a proton and electron are brought near each other.

3 Marks Questions

Q 1. Does work depends on frame of reference? Explain with suitable examples.

Sol. With change of frame of reference (inertial), force does not change while displacement may change. So the work done by a force will be different in different frames.

Example: If a porter with a suitcase on his head moves up a staircase, work done by the upward lifting force relative to him will be zero (as displacement relative to him is zero) while relative to a person on the ground will be mgh.

Commonly Made Error

- Most of the students make errors considering work is always same.

Answering Tip

- Take care of reference frame while calculating work.

Q 2. Define energy. Give its units and dimensions.

Sol. The energy of a body is defined as its capacity of doing work.

(i) Since energy of a body is the total quantity of work done, therefore it is a scalar quantity.

(ii) Dimension: $[ML^2T^{-2}]$, it is same as that of work or torque.

(iii) Units: Joule [S.I.], erg [C.G.S.]

Practical units: Electron volt (eV), Kilowatt hour (kWh), Calories (cal)

5 Marks Questions

Q 1. Derive the expression for elastic potential energy. AI

Sol. When a spring is stretched or compressed from its normal position ($x = 0$), work has to be done by external force against restoring force.

$$\vec{F}_{ext} = -\vec{F}_{restoring} = k\vec{x}$$

Let the spring is further stretched through the distance dx, then work done

$$dW = \vec{F}_{ext}.\vec{dx} = \vec{F}_{ext}.dx\cos 0° = kx\,dx \qquad [\text{As } \cos 0° = 1]$$

Therefore, total work done to stretch the spring through a distance x from its mean position is given by

$$W = \int_0^x dW = \int_0^x kx\,dx = k\left[\frac{x^2}{2}\right]_0^x = \frac{1}{2}kx^2$$

This work done is stored as the potential energy in the stretched spring.

$$\therefore \text{ Elastic potential energy } = U = \frac{1}{2}kx^2$$

$$U = \frac{1}{2}Fx \qquad \left[\text{As } k = \frac{F}{x}\right]$$

$$U = \frac{F^2}{2k} \qquad \left[\text{As } x = \frac{F}{k}\right]$$

$$\therefore \text{ Elastic potential energy } U = \frac{1}{2}kx^2 = \frac{1}{2}Fx = \frac{F^2}{2k}$$

Q 2. Explain different types of collisions in detail. Give examples to support your claim.

Sol.

Perfectly elastic collision	Inelastic collision	Perfectly inelastic collision
If in a collision, kinetic energy after collision is equal to kinetic energy before collision, the collision is said to be perfectly elastic.	If in a collision kinetic energy after collision is not equal to kinetic energy before collision, the collision is said to inelastic.	If in a collision two bodies stick together or move with same velocity after the collision, the collision is said to be perfectly inelastic.

Coefficient of restitution $e = 1$	Coefficient of restitution $0 < e < 1$	Coefficient of restitution $e = 0$
$(KE)_{final} = (KE)_{initial}$	Here kinetic energy appears in other forms. In some cases $(KE)_{final} < (KE)_{initial}$ such as when initial KE is converted into internal energy of the product (as heat, elastic or excitation) while in other cases $(KE)_{final} > (KE)_{initial}$ such as when internal energy stored in the colliding particles is released.	The term 'perfectly inelastic' does not necessarily mean that all the initial kinetic energy is lost, it implies that the loss in kinetic energy is as large as it can be. (Consistent with momentum conservation).
Examples : (1) Collision between atomic particles (2) Bouncing of ball with same velocity after the collision with earth.	*Examples :* (1) Collision between two billiard balls. (2) Collision between two automobiles on a road. In fact all majority of collisions belong to this category.	*Example:* Collision between a bullet and a block of wood into which it is fired, when the bullet remains embedded in the block.

Numericals

Q 1. A force acts on a 30 g particle in such a way that the position of the particle as a function of time is given by $x = 3t - 4t^2 + t^3$, where x is in metres and t is in seconds. What is the work done during the first 4 seconds ?

Sol. $v = \frac{dx}{dt} = 3 - 8t + 3t^2$

$\therefore v_0 = 3$ m/s and $v_4 = 19$ m/s

$W = \frac{1}{2}m(v_4^2 - v_0^2)$ (According to work energy theorem)

$= \frac{1}{2} \times 0.03 \times (19^2 - 3^2) = 5.28\text{J}$

Q 2. A force of 5 N acts on a 15 kg body initially at rest. What is the work done by the force during the first second of motion of the body ?

Sol. $W = Fs = F \times \frac{1}{2}at^2$ $\quad \left[\text{from } s = ut + \frac{1}{2}at^2\right]$

$\Rightarrow W = F\left[\frac{1}{2}\left(\frac{F}{m}\right)t^2\right] = \frac{F^2t^2}{2m} = \frac{25 \times (1)^2}{2 \times 15} = \frac{25}{30} = \frac{5}{6}\text{J}$

Q 3. A force of 5 N, making an angle θ with the horizontal, acting on an object displaces it by 0.4 m along the horizontal direction. If the object gains kinetic energy of 1 J, what is the horizontal component of the force ?

Sol. Work done on the body = K.E. gained by the body

$Fs\cos\theta = 1 \text{ Joule} \Rightarrow F\cos\theta = \frac{1}{s} = \frac{1}{0.4} = 2.5 \text{ N}$

Q 4. Two bodies A and B having masses in the ratio of 3 : 1 possess the same kinetic energy. What is the ratio of their linear momentum ?

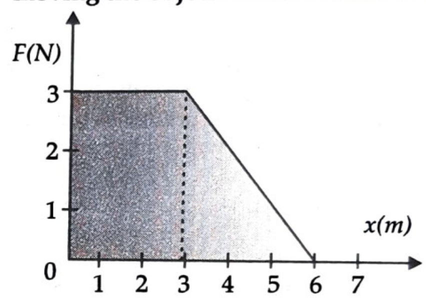
Sol. $p = \sqrt{2mE}$ $\therefore \frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}}$ (if E = constant)

$\therefore \frac{p_1}{p_2} = \sqrt{\frac{3}{1}}$

Q 5. A weight lifter lifts 300 kg from the ground to a height of 2 meter in 3 second. What is the average power generated by him ?

Sol. $P = \frac{\text{Work done}}{\text{time}} = \frac{mgh}{t} = \frac{300 \times 9.8 \times 2}{3} = 1960 \text{ W}$

Q 6. A force F acting on an object varies with distance x as shown here. The force is in newton and x in metre. What is the work done by the force in moving the object from $x = 0$ to $x = 6$ m ?



Sol. Work done = Area enclosed by F – x graph

= area of square + area of triangle

$= 3 \times 3 + \frac{1}{2}(3 \times 3) = 13.5 \text{ J}$

So the work done is 13.5 J.

Q 7. Two particles of masses m_1 and m_2 in projectile motion have velocities $\vec{v_1}$ and $\vec{v_2}$ respectively at time $t = 0$. They collide at time t_0. Their velocities become $\vec{v'_1}$ and $\vec{v'_2}$ at time $2t_0$ while still moving in air. What is the value of $\left|\left(m_1\vec{v'_1} + m_2\vec{v'_2}\right) - \left(m_1\vec{v_1} + m_2\vec{v_2}\right)\right|$? AI

Sol. The momentum of the two–particle system, at $t = 0$ is $\vec{P_i} = m_1\vec{v_1} + m_2\vec{v_2}$

Collision between the two does not affect the total momentum of the system.

A constant external force $(m_1 + m_2)g$ acts on the system.

The impulse given by this force, in time $t = 0$ to $t = 2t_0$ is $(m_1 + m_2)g \times 2t_0$

$\therefore$ |Change in momentum in this interval

$= \left|m_1\vec{v'_1} + m_2\vec{v'_2} - \left(m_1\vec{v_1} + m_2\vec{v_2}\right)\right| = 2(m_1 + m_2)gt_0$

So, the answer for the above question is $2(m_1 + m_2)gt_0$

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